

GRAPH THEORY

Tutorial – 9

- 1. Prove that every graph G has a vertex ordering relative to which greedy coloring uses $\chi(G)$ colors.**
- 2. Prove that a graph G is 2^k -colorable if and only if G is the union of k bipartite graphs.**
- 3. Prove that $\chi(G) = \omega(G)$ when \bar{G} is a bipartite graph without any isolated vertex.**

- 4. Let G be a graph whose odd cycles are pairwise intersecting, meaning that every two odd cycles in G have a common vertex. Prove that $\chi(G) \leq 5$.**

- 5. Prove that every k -chromatic graph has at least ${}^k C_2$ edges. Use this to prove that if G is the union of m complete graphs of order m , then $\chi(G) \leq 1 + m\sqrt{m-1}$.**